Wave-Mean Flow Interaction on Tidally Locked Planets

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Tidally locked planets with atmospheres have a global circulation with a superrotating eastward equatorial jet, and a hot-spot shifted eastwards from their substellar point. We used the shallow-water equations linearised about this eastward flow to show that the effect of the flow explains the form of the global circulation, particularly the hot-spot shift and the positions of the cold spots on the night-side.

1 - Tidally Locked Atmospheres

Tidally locked planets always present the same face to the star they orbit.

Observations of such planets show that their hottest part is shifted up to ten degrees eastward of the substellar point, and that their night-sides are warmer than expected from radiative equilibrium.

This implies a global circulation from day-side to night-side. GCM simulations show the atmospheres on such planets have an eastward superrotating jet, and a hot-spot shifted east of the substellar point.

Figure 1 shows the stationary response to a forcing which is sinusoidal in longitude, for the one-layer shallow-water equations linearised about zero background flow (Matsumo 1966).

This systems predicts an eastward equatorial acceleration, so it predicts the equatorial jet on these planets (Showman & Polvani 2011). However, the hot and cold spots are in the wrong place.

2 - Effect of Shear Flow

We linearised the one-layer shallow-water equations on an equatorial beta-plane about a zonally uniform shear flow $U(y)$ and a height perturbation $h(y)$ due to geostrophic balance. This is more accurate than our approximations above.

Our aim was to build on previous work by including the effect of the eastward equatorial jet on tidally locked planetary atmospheric dynamics.

The previous results in Showman and Polvani (2011) (linearised about zero flow) and Tsai et al. (2010) (linearised about uniform flow and height) have the same analytic solutions as in Matsumo (1966).

Our new system has no analytic solution, so we solve it with a pseudo-spectral method. We solve the equations:

$$ \begin{align*}
\partial_t u + \partial_y v &= (\alpha + \beta_0) u - \beta_1 v + \frac{1}{2} \nabla^2 u - \frac{1}{2} \nabla^2 v, \\
\partial_t v + \alpha v &= \beta_0 v - \beta_1 u + \frac{1}{2} \nabla^2 v - \frac{1}{2} \nabla^2 u, \\
\partial_t h + \alpha h &= \beta_0 h - \beta_1 u + \frac{1}{2} \nabla^2 h - \frac{1}{2} \nabla^2 h.
\end{align*} $$

We express the solution as a series of Hermite functions which correspond to the free modes of the system. A non-zero flow $U(y)$ and height $h(y)$ excite none of these free modes in the stationary response.

Figure 5 shows the effect of the shear flow on the meridional (real) parts of the forced solution, and Figure 6 shows its effect on the total forced solution on the beta-plane.

The beta-plane does not represent the latitude of the planet realistically, so we also solved the following equations for the same system on a sphere:

$$ \begin{align*}
\partial_t u + \partial_y v &= \nabla^2 v + \beta_1 u - \beta_0 v, \\
\partial_t v + \alpha v &= \beta_0 v - \beta_1 u + \nabla^2 u - \nabla^2 v, \\
\partial_t h + \alpha h &= \beta_0 h - \beta_1 u + \nabla^2 h - \nabla^2 h.
\end{align*} $$

We expressed this solution as a series of Legendre polynomials. Figure 7 shows the resulting solution, which now corresponds to a latitude-longitude field.

3 - Comparing to Simulations

Figure 8 shows the mid-atmosphere temperature and wind fields for a tidally locked planet with a 10 day period. It also shows the streamfunction and winds of the same simulation run with the jet on, showing the eastward Doppler-shift discussed above.

The shallow-water model predicts a large scale zonal symmetric circulation for rapidly forced or rapidly rotating planets. Figure 9 shows how our GCM results agree with this prediction.

Figure 10 shows the zonally averaged component of the mid-atmosphere pressure field, as the model spins up. The Rossby wave shifts from west to east as predicted.

Figure 11 shows the longitude of the maximum of this zonally averaged response, and the mean eastward flow speed. As the flow increases, the wave moves east.

4 - Conclusions

We linearised the shallow-water equations about an equatorial jet and its associated height perturbation, in a single-layer model of a tidally locked planet.

The solutions successfully predicted the behaviour of simulations of such atmospheres, particularly the shift of the wavenumber-1 standing wave as the equatorial jet forms.

Figure 12 summarises our work, showing how linearising the shallow-water model about the eastward jet explains the form of the circulation.

References